1 stepped pressure equilibrium code : sc00ac

1. Enforces spectral condensation constraint.
2. The representation of a given toroidal surface by Fourier harmonics is not unique. Any Fourier representation implies a particular angle parametrization, and different choices of angles results in a different Fourier representation. This freedom may be exploited to obtain a preferred angle parameterization.
3. A given toroidal surface may be described by $R(\theta,\zeta)=\sum_i R_i \cos\alpha_i$ and $Z(\theta,\zeta)=\sum_i Z_i \sin\alpha_i$, where $\alpha_i \equiv m_i\theta - n_i\zeta$. Intuitively, better numerical accuracy will be obtained at a given Fourier resolution the angle parameterization can be adjusted to condense the Fourier spectrum. Consider the spectral width,
$$M = \frac{1}{2}\sum_i c_i (R_i^2 + Z_i^2), \tag{1}$$
where c_i are constants, e.g. $c_i = m_i^p + n_i^q$ for integers p and q. As the angle parameterization is defined by the Fourier representation (and vice versa), the task is to vary the R_i and Z_i in order to minimize M;
4. The variation of the R_i and Z_i must be restricted by the constraint that the surface geometry does not change (the geometry of the surfaces will be adjusted in order to satisfy force balance). To minimize M we restrict attention to *tangential* variations of the form
$$\begin{aligned} \delta R &= R_\theta \, \delta u, && (2)\\ \delta Z &= Z_\theta \, \delta u, && (3) \end{aligned}$$
where δu is an arbitrary function.
5. The variation in the Fourier harmonics is given
$$\begin{aligned} \delta R_i &= \oint\!\!\oint d\theta d\zeta \;\; R_\theta \, \delta u \cos\alpha_i, && (4)\\ \delta Z_i &= \oint\!\!\oint d\theta d\zeta \;\; Z_\theta \, \delta u \sin\alpha_i. && (5) \end{aligned}$$
6. The first variation in M is given
$$\delta M = \oint\!\!\oint d\theta d\zeta \;\; (R_\theta X + Z_\theta Y)\,\delta u, \tag{6}$$
where $X \equiv \sum_i c_i R_i \cos\alpha_i$ and $Y \equiv \sum_i c_i Z_i \sin\alpha_i$. The Euler-Lagrange equation for the minimization of the spectral width, M, with respect to arbitrary variations, δu, is $I \equiv R_\theta X + Z_\theta Y = 0$.
7. The condition $I \equiv R_\theta X + Z_\theta Y = 0$ allows R to be expressed as a function of Z. Expanding the Fourier summation, we find
$$\begin{aligned} I &= \sum_{j,k} \left[(-m_j) R_j \sin\alpha_j \; c_k R_k \cos\alpha_k + (+m_j) Z_j \cos\alpha_j \; c_k Z_k \sin\alpha_k\right] && (7)\\ &= \sum_{j,k} \frac{m_j c_k}{2} \left\{ -R_j R_k \left[\sin(\alpha_j-\alpha_k)+\sin(\alpha_j+\alpha_k)\right] + Z_j Z_k \left[\sin(\alpha_k-\alpha_j)+\sin(\alpha_k+\alpha_j)\right]\right\}. && (8) \end{aligned}$$
8. The R_j may be determined (numerically) by setting the Fourier harmonics, I_i, to zero, where $I = \sum_i I_i \sin\alpha_i$ and the Z_j are assumed given.

1.0.1 Some double angle formulae for convenience

1.
$$\begin{aligned} 2\,\sin\alpha\,\sin\beta &= \cos(\alpha-\beta)-\cos(\alpha+\beta), && (9)\\ 2\,\sin\alpha\,\cos\beta &= \sin(\alpha-\beta)+\sin(\alpha+\beta), && (10)\\ 2\,\cos\alpha\,\cos\beta &= \cos(\alpha-\beta)+\cos(\alpha+\beta), && (11) \end{aligned}$$

sc00ac.h last modified on 2012-05-08 ;